Rational, Irrational Numbers

Rational and irrational numbers make up the set of real numbers.

Rational numbers are numbers that can be expressed in the form , where b is not equal to zero, and both a and b are integers. These, then, include all fractions and decimals that are either terminating (has an end), or nonterminating but repeating (usually can be expressed as a fraction).

Irrational numbers, on the other hand, include all nonterminating, nonrepeating numbers when expressed in decimal form. These usually do not have a fixed fraction form because there is no end to their numbers.

Take pi () as an example of an irrational number.

* Pi does not have a known endpoint to its decimals, therefore, it is nonterminating.
* Pi does not have a pattern to its digits, therefore, it is nonrepeating.
* 3.14 and are merely approximations for the true value of pi. Since it does not have an endpoint, and it does not have a pattern, we cannot really express the true value of pi as a fraction.

Fractions

Fractions are numbers that follow the format , where b is not equal to zero, and both a and b are integers. The variable a here represents the numerator, and the variable b represents the denominator. They are separated by a fraction bar, which denotes division.

Several kinds of fractions:

1. Proper fractions

* Fractions where the numerator is less than the denominator.

1. Improper fractions

* Fractions whose numerator is greater than or equal to the denominator.

1. Mixed number

* A combination of whole number and proper fraction, in the form of , which is, simply, .

Simplifying fractions is done when the numerator and denominator can still be reduced to its lowest terms. When the numerator and denominator are relatively prime, though, there is no need to simplify. One can simplify a fraction by dividing both the numerator and denominator by their highest common factor.

*Addition and Subtraction of Fractions*

As long as all operations are addition, and the denominators are common to all fraction, you can do the addition all at the same time.

*Multiplication of Fractions*

As long as all operations are multiplication, you can do multiplication or simplification in any order.

*Simplification of Fractions*

You can simplify fractions in multiplication horizontally (simplify fraction itself) or diagonally (when one sees a numerator of one fraction that has a common factor with the denominator of another fraction, you can cancel them out with their HCF).

*Division of Fractions*

As seen here, in order to divide, one must get the reciprocal of the fraction dividing the other fraction. The reciprocal, or multiplicative inverse, is what you multiply to a certain number in order to get 1.

*Positive and Negative Fractions*

Positive fractions are obtained from fractions whose numerator and denominator *have the same sign*. (When both are negative, though, it can still be simplified into a fraction with positive numerator/denominator) Examples of these would be and , simplified to become .

Negative fractions are obtained from fractions whose numerator and denominator *have a different sign*. (When the denominator is negative, the negative sign can be transferred to the numerator, or to the fraction in general) Examples of these would be and simplified to become *or* .

Comparison of Rational Numbers

You can compare rational numbers either graphically or algebraically.

Graphically, you can illustrate this comparison using a number line. Just remember that numbers to the right are greater than the ones to the left. All numbers to the left of zero are negative rational numbers while all numbers to the right of zero are positive rational numbers.

Algebraically, one can also compare rational numbers. You must also remember the following:

1. The denominator of the fractions should always be positive. This is to be in line with what is defined to be a fraction in simplest form.
2. When the denominator is the same, you should only compare the numerators. When the denominators are different, though, make sure that the fractions have the same denominator before comparing the numerators.
3. When the fractions to be compared have the same numerator but different denominator, the one with the smaller denominator is the greater one.
4. For numbers who are both positive or negative, the denominator should be the same first. Comparing positive numbers, the one with a greater numerator is the greater of the two, but for both negative numbers, the one with the smaller numerator is the greater of the two.
5. Comparing zero and a positive or negative number is simple. Positive numbers are greater than zero; negative numbers are less than zero.

Decimal Operations

*Addition and Subtraction*

In addition and subtraction of decimals, you should treat it like addition and subtraction of integers. The only difference here is that you need to align the decimal points of the two addends. This is important so as not to confuse the place values with one another.

*Multiplication and Division*

Unlike addition and subtraction, you do not need to align the decimal points anymore. However, there are some different ways of multiplying and dividing.

For *multiplication*, do not mind the decimal places while multiplying. Only count the number of decimal places each factor has in the end. The sum of the amount of decimal places each factor has will determine how many decimal places the final product will have.

For *division*,

Can be expressed as

Upon rearranging the decimal numbers, you may multiply both numerator and denominator by factors of ten, to reach the point where the denominator is and integer.

When the decimals are gone, you may now divide according to the rule of division of integers.

Set Notation, Kinds of Sets

A group/collection of numbers, things, people, places or any other individual pieces of data is called a set. Anything belonging to a set is called an element of the set. A set is well-defined if its elements are listed specifically or if its elements are described in a way that makes it possible to decide whether an object is or is not an element of the set.

The same element is not listed more than once in a set. The order of elements in a set is not important. Capital letters are often used to represent sets, while small letters are used to represent the elements of the set.

A set is described using the following:

:= means “is defined as”.

An element of a set is described using the following:

“a is an element of set X”

“b is not an element of set X”

A set containing all elements being considered is called a universal set, denoted by the capital letter U. A set with only one element is called the unit set. A set with no elements is called an empty set or null set. It is denoted by the symbol { } or .

The number of elements in a set is called the cardinal number (or cardinality) of the set, denoted as n( ). A set is finite if it is an empty set or if all its elements can be listed. It is then infinite if it has a limitless number of elements.

Two of the most common methods of describing sets are the **roster** or **listing** method and the **rule method** or **set-builder notation**.

To describe a set using the roster method, the elements of a set are listed, separated by commas, and enclosed within a pair of braces.

E = {x, y, z}

To describe a set using the rule method, the elements of a set are represented by a small letter followed by a vertical bar, read as “such that” and a description enclosed within a pair of braces.

E = {a|a is one of the (description of set)}

Two sets A and B are considered equal (A=B) or identical if and only if they have exactly the same elements. Two sets A and B are equivalent (A~B) if there is a one-to-one correspondence between their elements, though the elements need not be exactly the same.

A set A is a subset of another set B () if every element of set A is also an element of set B. This can also mean that the two sets can be equal.

A set A is a proper subset of set B () if every element of A is also an element of B and B has at least one element not in set A.

A set A is an improper subset if set B if set A is a subset of B and A=B.

The set of all subsets of a set is called the power set of the set, denoted P( ). The cardinality of the power set of a finite set is given by the formula 2r where r is the cardinality of the finite set.

Set Operations

Four set operations exist: union, intersection, complement, and difference.

1. Union ()

This is used to signify a combination of elements in two sets A and B. This includes elements in A, elements in B, and elements in A and B (if they overlap).

A union B is written as so:

1. Intersection ()

This indicates common elements between two sets A and B. A intersection B will give the same result as B intersection A. The result of finding the intersection of a null set and the universal set U is a null set.

A intersection B is written as so:

1. Complement (‘ or – or c)

This describes the elements of a set A which is not in a set B. The complement of an identical set is a null set. Also, the complement of a universal set is a null set.

A complement is written as so:

1. Difference (-)

The difference of a set A and another set B can also be defined as A intersection B complement. It is found by getting the portion of the universal set that does not include the certain set B but at the same time includes the set A.

A difference is written as so: